An Optimisation Model for the Management of Alluvial Rivers

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ABSTRACT: *A model is developed to interpret and predict the response of alluvial rivers to changes in runoff, sediment supply or bank stability conditions. The approach is based on an equilibrium analysis and uses an optimisation formulation to determine the steady-state or "regime" hydraulic geometry for a given set of input variables. The model is useful as a management tool and can be used to assess the effects of proposed land use changes or engineering projects on the geometry of alluvial rivers.*

1. INTRODUCTION

Modelling the hydraulic geometry of alluvial rivers is of interest to hydraulic engineers, hydrologists and environmental managers engaged in assessing the future response of the channel to catchment development and land-use changes. Changes in the imposed discharges, sediment load or the ability of the banks to withstand erosion can result in adjustment of the channel boundary, and the development of a new hydraulic geometry. The channel adjustments may result in changes that are determined to be undesirable for economic, environmental, or aesthetic reasons and include bank erosion and loss of riparian habitat and land adjacent to the river; degradation of the channel bed which can undermine bridge foundations and other hydraulic structures; aggradation of the channel bed which can reduce the channel capacity and result in an increased frequency of over-bank flooding; and changes in the physical nature of the channel that may impact the aquatic habitat.

In this paper an analytical model will be described. The basic goal of the model is to calculate the equilibrium or "regime" hydraulic geometry of alluvial rivers for a given set of inputs. The model is presented as an optimisation formulation. Optimisation models have been developed previously by Chang (1980), Yang *et al.* (1981), and White *et al.* (1982) among others to predict the geometry, or aspects of the geometry, of alluvial channels. The model is an extension of this earlier work.

In the companion paper (Millar and Quick, 1996b), the model is used to demonstrate the influence of the bank stability and riparian vegetation on the channel geometry.

2. DEFINITION OF EQUILIBRIUM

It is an underlying assumption of equilibrium channel analysis that alluvial rivers develop a mean hydraulic geometry in response to the water discharge, sediment load, and sediment properties that are determined by the geology and hydrologic regime of the upstream areas. Blench (1969, p. 1) refers to the "basic principle of self-adjustment" and states that:

> The fundamental fact of river science, pure and applied, is that regime channels tend to adjust themselves to average breadths, depths and slopes and meander sizes that depend on (i) the sequence of water discharges imposed on them, (ii) the sequence of sediment discharges acquired by them from the catchment erosion, erosion of their own boundaries, or other sources and (iii) the liability of their cohesive banks to erosion or deposition.

The term regime used by Blench is considered to be synonymous with alluvial. Alluvial rivers are by definition those that have bed and banks composed of sediment that is actively transported by the river.

The optimisation model is based on an analysis of this equilibrium state. Transient adjustments are not considered. A channel is defined herein as being in equilibrium when the following conditions are satisfied:

1. The mean hydraulic geometry of the representative channel reach remains unchanged over an appropriate time scale for which a steady-state equilibrium can be assumed.

2. There is no net erosion or deposition along the reach.

3. Any perturbations from the equilibrium geometry will be offset, and the equilibrium restored.

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Note that this definition applies to reach-averaged, and not local values of the hydraulic geometry. The term steady-state refers to a time-invariant equilibrium. The absolute time period over which this approximation is valid will depend upon the system being studied. It is valid where there are no significant or systematic changes in the mean annual characteristics of the discharge and sediment load, or in the competence of the bank sediment. In general the steady-state approximation may apply over engineering time scales up to 100 years or so.

3.0 MODEL BASIS

An alluvial channel is a relatively complex system with perhaps up to seven degrees of freedom (Hey, 1978). The system is indeterminate in that there are more unknown dependent variables or degrees of freedom than there are equations for solution. Even for a simplified channel geometry with only three degrees of freedom: width, depth, and slope, the solution remains indeterminate as there are only two relations, flow resistance-continuity, and sediment transport, available for solution. Being indeterminate there are an infinite number of solutions that satisfy the available governing equations.

The basis of the optimisation model is the assumption that the equilibrium or regime hydraulic geometry corresponds to an optimum configuration. This assumption allows a single, unique solution to be selected from the infinite possibilities.

3.1 Optima in Fluvial Systems

The presence of an optimum geometry in fluvial hydraulics was first demonstrated by Gilbert in 1914. In flume experiments under conditions of fixed discharge rate and channel slope, it was demonstrated that by varying the width of the flume an optimum value exists where the sediment transporting capacity of the flume was a maximum.

The mechanisms responsible for the optimum in Gilbert's experiments can be readily explained. For narrow flume widths much of the shear force is acting on the side walls, and this, together with the narrow bed widths over which sediment transport can occur, results in a low total transport rate. Conversely for the large flume widths the depth of flow and the bed shear stress both become small, and hence the total sediment transport rate also becomes small. Between these two extremes lies an optimum where the sediment transport rate is maximised.

Gilbert's experimental result can be duplicated numerically. An example of a solution curve is shown schematically in Figure 1(a). The channel slope and discharge are constant for all points on the solution curve. In Figure 1(b) a solution curve is shown where the channel slope is a variable, and the discharge and sediment transporting capacity are now fixed. In this case the optimum is the point where the slope is a minimum. This second case is considered to be analogous to most natural alluvial rivers over engineering time scales. The discharge and sediment load are imposed, and the width, depth and channel slope are dependent variables that develop in response to these imposed values.

The experimental results of Gilbert (1914) together with the numerical analyses discussed above combine to lend significant support for the actual existence of an optimum in the fluvial system. The fundamental assumption employed in the optimisation model is that a natural river channel will tend to adjust to this optimum, and that this optimum corresponds to the equilibrium hydraulic geometry.

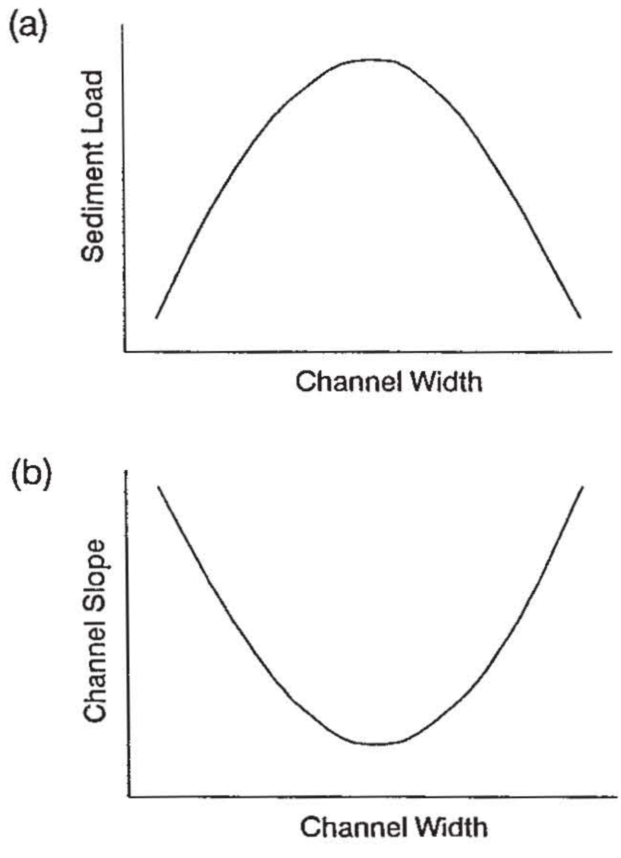

Figure 1. Schematic representation of the optimum in fluvial systems. (a) Constant discharge and slope. (b) Constant discharge and sediment load.

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4.0 MODEL FORMULATION

The formulation of the optimisation model will now be discussed.

4.1 Independent Variables

The independent variables represent the known, external controlling variables which are inputs to the system. The variables which are considered independent include the physiographic, geologic and hydrologic properties of the catchment. These are the geology, relief, climate, runoff, vegetation, and the volume and calibre of the sediment yield.

Additional independent variables are related to the channel boundary, namely the bank vegetation, and bank sediment parameters such as cohesion and friction angle. The valley slope is also considered to be independent over engineering time scales. Although the valley slope can be considered to be a dependent variable over geologic time scales, any significant change in the valley slope requires the removal or deposition of large volumes of alluvium which can only occur over long periods of time.

The channel is modelled using the steady-state, mean values of the sediment yield and flow-duration data or a single representative discharge, as well as the mean bank stability parameters for the representative channel reach.

4.2 Dependent Variables

The dependent variables are the unknown channel geometry variables: width, depth, slope, bank angle, roughness, velocity, sinuosity, meander and pool-riffle wavelength, meander radius of curvature, and bedforms. In the current model a simplified channel geometry is assumed. Secondary currents and the planform variables will not be considered explicitly, however a comparison of the channel and valley slopes gives a measure of the channel sinuosity and therefore some limited information on the planform geometry.

4.3 Constraints

The principal constraints in the model are the discharge, sediment load, bank stability, and valley slope constraints.

4.3.1 Discharge Constraint

The discharge is a constraint on the channel system as it represents virtually the total energy input. The channel development and sediment transport must be accomplished with the available flows. The discharge constraint demands that the channel has a discharge capacity equal to the imposed value of the bankfull discharge. The bankfull discharge is considered to be an independent variable.

The model requires a flow resistance equation together with continuity to satisfy the discharge constraint. The discharge constraint 'sizes' the channel. The channel must have a discharge capacity equal to the bankfull discharge, Q_{bf}:

$$U\,A = Q_{bf} \qquad (1)$$

where U = the mean velocity, and A = the cross-sectional area of the flow. The value of U is obtained from either the Darcy-Weisbach or Manning equations.

4.3.2 Sediment Load Constraint

The sediment load constraint is a key component of the model and ensures that the amount of sediment that is imposed at the upstream end of the channel is transported without net deposition or scour. In other words the sediment load constraint demands that sediment continuity is maintained along the channel reach:

$$P_{bed}\,g_b = G_b \qquad (2)$$

where P_{bed} = bed width or perimeter, g_b is the sediment transport rate per unit channel width, and G_b is the imposed sediment load.

The value of g_b is calculated using a suitable sediment transport relation that is based on shear stress. Sediment transport relations containing a critical shear stress that must be exceeded before sediment transport commences cannot be used. The optimisation scheme is unable to handle zero transport rates.

4.3.3 Bank-Stability Constraint

A requirement of equilibrium channels is that the banks must be stable. Note that stability here applies to the reach-averaged condition of the banks. Locally, such as along the outer bend of a meander, the banks may not be stable. However the reach-average condition of the banks must be one of stability, otherwise there would be a net change in the hydraulic geometry over time, and the channel could not be considered to be in equilibrium.

The bank-stability constraint requires that the reach averaged condition of the banks be stable. The bank-stability constraint can be formulated for banks with noncohesive sediment or cohesive sediment,

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and is discussed in some detail in the companion paper (Millar and Quick, 1996b).

4.3.4 Valley-Slope Constraint
The valley slope forms an additional constraint or physical bound to the solution. The valley-slope constraint requires that the equilibrium channel slope is less than or equal to the valley slope:

$$S \leq S_v \qquad (3)$$

where S is the channel slope, and S_v is the valley slope. S_v is considered to be an independent variable.

4.4 Objective Function

The objective function in this formulation is the maximisation of the coefficient of sediment transport efficiency, η, which was developed from Bagnold's (1966) streampower arguments (Millar, 1994). The value of η is defined as:

$$\eta = \frac{G_b}{\rho Q S} \qquad (4)$$

Where G_b = the dry sediment transport rate in mass units, and ρ = the density of water, Q = representative discharge and may be the bankfull or mean annual discharge, and S = the channel slope (uniform flow is assumed). The product $\rho\, Q\, S$ is the total stream power in mass units.

Note that maximization of η is equivalent to the extremal hypotheses of maximization of sediment transport capacity (White *et al.* 1982, Millar and Quick, 1993) under conditions of fixed Q and S, and is equal to the minimisation of streampower (Chang, 1980) or minimum S for imposed values of G_b and Q. In the complete form of (4) which is not shown here, maximization of η is also equivalent to the minimisation of the energy dissipation rate of Yang and Song (1979).

The optimum solutions are shown schematically in Figure 1 (a & b). In Figure 1(a), under conditions of fixed Q and S, the optimum corresponds to a maximum value of G_b. In Figure 1(b) G_b and Q are fixed, and the optimum corresponds to a minimum value of S.

5.0 OPTIMISATION SCHEME

A computer program was developed to determine the optimum hydraulic geometry which satisfies the discharge, bedload transport and bank-stability constraints. The model uses a heuristic, iterative search routine to locate the optimum. However any non-linear optimisation technique could be used.

No evidence of local or multiple optima has been detected to date.

The optimisation model can be formulated in a number of ways depending upon the available data and the objectives. In the simplest version, the sediment transporting capacity of the channel is modelled at the bankfull discharge only. A more complex version requires the entire flow-duration curve to be discretised and used as input, and the sediment transporting capacity is modelled for the entire range of flows.

The model can be formulated as a fixed-slope model where the channel slope is held constant, and the optimum cross-section is determined by varying the channel width. The optimum in this instance corresponds to the maximum transporting capacity. Alternatively in the variable-slope formulation the imposed sediment load is fixed as an independent variable, and the channel slope is free to adjust to the optimum. In this instance the optimum corresponds to the minimum channel slope.

The variable-slope model is generally used to model natural rivers. However the fixed-slope model is useful when estimating or calibrating the bank-stability parameters. This is discussed in the companion paper (Millar and Quick, 1996b).

A version of the variable slope model is presented in greater detail in Millar and Quick (1993). The full range of models is discussed in Millar (1994).

6.0 MODEL APPLICATION

The optimisation model can be used to assess the response of a channel to any situation where there is alteration in the volume or size distribution of the imposed sediment load, the volume and timing of the flows, or the properties of the bank sediment. For example the construction of a dam will affect the sediment supply and flows. The sediment supply will be reduced dramatically, often effectively to zero directly downstream from the dam. The flows are usually affected to a large extent, typically by truncating the higher flows, and increasing the proportion of low flows. The total runoff volumes may or may not be affected. The bank stability parameters may not be affected.

Other potential applications relate to land-use changes. For instance removal of forest cover by logging or for agricultural development may increase the sediment yield from the catchment, and yet may or may not affect the runoff to any great extent. Often the riparian vegetation is affected by these developments, and as is demonstrated in the companion paper (Millar and Quick, 1996b), this can have a profound influence on the bank stability.

Urbanisation can severely alter the runoff response and sediment yield from affected areas of the catchment.

The model can be used to perform sensitivity analyses to determine the effect of a potential development on the river channel where the post-development inputs cannot be accurately determined. For instance to assess the response of a river channel to increased sediment load, the model can be initially calibrated using the observed hydraulic geometry. A sensitivity analysis can then be performed to determine the sensitivity of the channel under the range of potential increases in the sediment load.

Regardless of the type of development or catchment disturbance, the input required for the model is a flow-duration curve or representative discharge (bankfull discharge), an estimate of the volume of sediment load and grain size distribution, and estimates of the bank stability parameters. Hydrologic modelling and sediment budget studies may be necessary, together with field observations, to determine the appropriate values to use as input to the model. These values may be difficult to measure in an intact system let alone to predict the values for a disturbed catchment. Estimates of the current sediment load of a stable river system can be obtained using the observed hydraulic geometry, together with the measured or estimated flow-duration curve.

When using the optimisation model it must be realised that the optimum value is only a theoretical value which the river may show a tendency to adjust towards. The optimisation model is only an adjunct to other techniques such as air photo interpretation, and field monitoring of observed channel adjustments of the river of interest, and in nearby channels that may have been subjected to similar disturbances. Any modelling results must be tempered with sound engineering judgement, and must recognise the location of geologic controls that may further constrain the system.

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